PG – 259

III Semester M.Sc. Examination, January 2018
(CBCS)
MATHEMATICS
M303T : Fluid Mechanics

Time : 3 Hours Max. Marks : 70

Instructions : 1) *Answer **any five** questions.*
2) ***All** questions carry **equal** marks.*

1. a) Define a Cartesian tensor of order 2 and show that each of the transformation rules follows from the other.
   b) State and prove divergence theorem for a tensor field $\underset{\sim}{A}$. **(8+6)**

2. a) Distinguish between :
      i) Lagrangian and Eulerian descriptions of motion.
      ii) Pathlines and streamlines.
   b) Obtain the expression for material derivative in the spatial form and hence obtain the formula for acceleration in components.
   c) Establish Reynolds transport formula. **(4+5+5)**

3. a) With usual notations, derive the continuity equation and hence show that
$$\frac{D}{Dt}\int_v \rho\phi dv = \int_v \rho\frac{D\phi}{Dt}dv.$$
   b) Derive the field equation for conservation of angular momentum. **(7+7)**

4. a) Distinguish between nonviscous and viscous fluids. Also, find the pressure distribution in an incompressible nonviscous fluid moving under the earth's gravitational field with velocity $\vec{q} = \text{grad}\left(x^3 - 3xy^2\right)$.
   b) Derive the Navier-Stokes equation for a compressible fluid. **(7+7)**

5. a) State and prove Kelvin's minimum energy theorem.
   b) Prove that $\frac{D}{Dt}\left(\frac{\vec{W}}{\rho}\right) = \left(\frac{\vec{W}}{\rho}\cdot\nabla\right)\vec{q}$ where quantities have their usual meaning. **(8+6)**

P.T.O.

6. a) Define a doublet. Obtain the complex potential for a doublet and find potential and stream functions.

   b) What arrangement of sources and sinks will give the complex potential

   $W = \log\left(z - \frac{a^2}{z}\right)$. Also find the streamlines. (7+7)

7. a) Obtain the velocity distribution for plane Poiseuille flow and find the maximum velocity.

   b) Explain Stokes's first problem and show that the velocity distribution for such a flow is $u(z, t) = U[1 - \text{erf}(n)]$, where quantities have their usual meaning. (7+7)

8. a) Starting from the vorticity transport equation for an incompressible viscous fluid, show that for an unsteady motion in circles with centres on the z-axis

   the said equation reduces to $\frac{\partial w}{\partial t} = \gamma \frac{\partial^2 w}{\partial r^2} + \frac{\gamma}{r}\frac{\partial w}{\partial r}$. Further, verify that

   $w = \frac{A}{t} e^{-r^2/4\gamma t}$ is the solution of this equation.

   b) Explain the energy dissipation due to viscosity and show that

   $W = \mu \int_v |\vec{w}|^2 \, dv - \mu \int_s (\vec{q} \times \vec{w}) \cdot \hat{n} ds$

   where the quantities have their usual meaning. (7+7)

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III Semester M.Sc. Degree Examination, Dec, 2013/Jan. 2014
(Scheme Y2K11 – RNS)
MATHEMATICS
M-304 : Fluid Mechanics

Time : 3 Hours Max. Marks : 80

Instructions : *i) Answer **any five** (5) questions choosing atleast (**2**) from Parts **A** and **B**, and **one** from Part **C**.*
*ii) **All** questions carry **equal** marks.*

PART – A

1. a) State and prove Kelvin's minimum energy theorem. **9**

   b) Derive the equation of impulsive motion and hence deduce that the impulsive pressure satisfies the Laplace equation in the absence of body force. **7**

2. a) Define complex potential and discuss the flow whose complex potential is given by
   $w = \frac{2f}{z}$ (f : constant). **8**

   b) Verify whether the complex potential $w = -m \ln(z + c) - m \ln(z - c)$ represents a flow that is an image system. **8**

3. a) Find the stream function and potential function of a doublet. **8**

   b) Obtain the complex potential of a doublet passing through z = a and whose axis makes an angle α with the positive direction of x-axis. **8**

PART – B

4. Obtain the velocity distribution for

   i) Generalized plane Couette flow

   ii) Hagen-Poiseuille flow. **16**

P.T.O.

5. a) With usual notation, show that

$$u = U - \frac{a}{4r^3}(3r^2 + a^2)U + \frac{3a}{4r^5}(a^2 - r^2)Ux^2$$

$$v = \frac{3a}{4r^5}(a^2 - r^2)Uxy$$

$$w = \frac{3a}{4r^5}(a^2 - r^2)Uxz$$

are the components of velocity for a slow and steady flow of an incompressible viscous fluid past a fixed rigid sphere. 14

b) Write a short note on Reynolds number. 2

6. a) Stating the assumptions made, show that the rate of energy dissipation due to viscosity is

$$W = \mu \int \omega^2 dv.$$ 5

b) Explain the concept of boundary layer. Establish Von-Karman's integral equation for a two-dimensional Prandtl boundary layer equation. 11

PART – C

7. a) Derive the governing equation for a simple turbulent flow of a Newtonian incompressible fluid with no body forces. 8

b) What is closure ? Explain with any one closure model. 8

8. a) Classify flows based on Mach and Reynolds numbers. 6

b) Derive the perfect gas equation in the standard form. 10

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III Semester M.Sc. Examination, December 2015
(CBCS)
MATHEMATICS
M303T : Fluid Mechanics

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **any five full** questions.*

1. a) Show that :

   i) If $a_{ij}b_j$ are components of a vector then a_{ij} are components of a tensor.

   ii) A tensor is orthogonal iff $\underset{\sim}{A}\vec{a} \cdot \underset{\sim}{A}\vec{b} = \vec{a} \cdot \vec{b}$ for all vectors $\vec{a}$ and $\vec{b}$. Also deduce that $\left|\underset{\sim}{A}\vec{a}\right| = |\vec{a}|$ for all vectors if $\underset{\sim}{A}$ is orthogonal. **(4+4)**

   b) State and prove divergence theorem for a tensor field $\underset{\sim}{A}$. **6**

2. a) Define : Path lines, stream lines and Vortex lines.

   If the acceleration is the gradient of a scalar function, then show that the circulation round a material curve remains constant in time t. **8**

   b) With usual notation, show that $\dfrac{D}{Dt}\int_v \phi\, dv = \int_v \dfrac{\partial \phi}{\partial t}\, dv + \int_s \phi\, \vec{q} \cdot \hat{n}\, dS$. **6**

3. a) Derive the equation of continuity in the form : $\dfrac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{q}) = 0$ and hence show that $\dfrac{D}{Dt}\int_v \rho\phi\, dv = \int_v \rho \dfrac{D\phi}{Dt}\, dv$, where the quantities have their usual meaning. **7**

   b) Using an appropriate conservation law show that the stress tensor is symmetric. **7**

P.T.O.

4. a) With usual notation derive Navier Stokes equation. 8

b) For a certain flow of a non-viscous fluid of constant density under the Earth's gravitational field, the velocity distribution is given by $\vec{q} = -\nabla\phi$, where $\phi = x^3 - 3xy^2$. Find the pressure distribution. 6

5. a) State and prove Kelvin's minimum energy theorem. 8

b) Define any two non-dimensional numbers and discuss their physical significance. 6

6. a) Discuss the flow whose complex potential is given by $W = Uz + m \, Ln\,(z - a) - m \, Ln\,(z + a)$. 7

b) Find the image system of a doublet. 7

7. State and prove Blasius theorem and any one of its major applications. 14

8. Obtain exact solution of the Navier-Stokes equation for the following problems :

a) Generalized Plane-Couette flow and

b) Stokes' first problem. (7+7)

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III Semester M.Sc. Examination, December 2016
(CBCS)
MATHEMATICS
M303 T : Fluid Mechanics

Time : 3 Hours Max. Marks : 70

*Instruction : Answer **any five** full questions.*

1. a) Define an isotropic tensor. If a_{ij} are components of an isotropic tensor then show that $a_{ij} = \alpha\delta_{ij}$ for some scalar α.
   b) State and prove Stokes's theorem for a tensor field $\underset{\sim}{A}$. **(8+6)**

2. a) Explain briefly :
      i) Continuum hypothesis.
      ii) Lagrangian and Eulerian descriptions of motion.
      iii) Path lines, stream lines and vortex lines.
   b) Establish the Reynolds transport formula and hence deduce the expression for the rate of change of a material volume. **(9+5)**

3. Derive the field equations for conservation of linear momentum and energy. **14**

4. a) Establish Euler's equation of motion.
   b) Find the pressure distribution for a velocity field $\vec{q} = k(x^2 - y^2)\hat{i} - 2kxy\hat{j}$ (k = constant) which satisfies the Navier-Stokes equation for an incompressible fluid in the absence of body force. **(6+8)**

5. a) Derive the Helmholtz vorticity equation and stating the assumptions made. Deduce that $\vec{W}/\rho$ = constant for a travelling fluid element.
   b) Define impulsive motion. Derive the general equation of impulsive motion and stating the conditions. Show that the impulsive pressure is harmonic. **(7+7)**

P.T.O.

6. a) Define : complex potential, source, sink and doublet. For a two-dimensional flow field given by $\psi = xy$, show that the flow is irrotational. Also, find the velocity potential, streamlines and potential lines.

   b) State and prove the Mitne-Thomson circle theorem. **(8+6)**

7. Obtain the velocity distribution for

   i) Generalised plane Couette flow.

   ii) Hagen-Poiseuille flow. **(7+7)**

8. a) Discuss the velocity distribution for Stokes's second problem by deriving an expression for the velocity field.

   b) Stating the assumptions made, show that the rate of energy dissipation due to viscosity of the fluid is $W = \mu \int_V w^2 \, dV$, where the quantities have their usual meaning. **(8+6)**

III Sem

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III

Second Semester M.Sc. Examination, June 2015
(RNS) (2011-12 and Onwards)
MATHEMATICS
M – 205 : Continuum Mechanics + Fluid Mechanics.

Time : 3 Hours   Max. Marks : 80

Instructions : *Answer* ***any five*** *questions, choosing atleast* ***one*** *from* ***each*** *Part.* ***All*** *questions carry* ***equal*** *marks.*

PART – A

1. a) For an arbitrary vector with components b_i, if $a_{ij}b_j$ are components of a vector then show that a_{ij} are components of a second order tensor. Hence, show that δ_{ij} are components of a second-order tensor. 6

b) Prove that $\underset{\sim}{A}$ is a second-order tensor iff it is a linear transformation on vectors and $a_{ij} = \hat{e}_i . \underset{\sim}{A}\hat{e}_j$. 5

c) If $\underset{\sim}{A}$ is an orthogonal tensor such that $\underset{\sim}{A}\vec{a} = \vec{a}$ for any vector $\vec{a}$, then show that $\underset{\sim}{A}^T\vec{a} = \vec{a}$ and the dual vector of skew $\underset{\sim}{A}$ is collinear with $\vec{a}$. 5

2. a) Define : gradient of a vector, divergence and curl of a tensor. 3

b) For $\vec{u} = x_1^2x_2\hat{e}_1 + x_2^2x_3\hat{e}_2 + x_3^2x_1\hat{e}_3$, verify the identity curl $\overline{\nabla u^T} = \nabla$ curl $\vec{u}$. 7

c) State and prove Stokes' theorem for a tensor field. 6

PART – B

3. a) Explain briefly the following :

i) Continuum hypothesis.

ii) Deformation of arc, surface and volume elements. 8

P.T.O.

b) For the deformation defined by the equations :

$x_1 = \alpha x_1^0 + \beta x_2^0, x_2 = -\alpha x_1^0 + \beta x_2^0, x_3 = \gamma x_3^0$ where α, β and γ are positive constants, find $\underset{\sim}{F}$, $\underset{\sim}{F}^{-1}$ and J. Is the deformation isochoric ? 5

c) Obtain an expression for Green strain tensor. 3

4. a) Obtain a formula for the material derivative in the spatial form. 3

b) Define : path lines and stream lines. Find these lines for the flow defined by the velocity field $\bar{v} = (1+at)\hat{e}_1 + x_1\hat{e}_2$ (a is a constant). Comment on these lines when $a = 0$. 7

c) Establish Reynolds transport formula and hence deduce that $\frac{DV}{Dt} = \int_V (\text{tr}\,\underset{\sim}{D})\, dV$. 6

5. a) Establish Cauchy's law in the form $\bar{s}(\hat{n}) = \underset{\sim}{T}^{T}\hat{n}$, where the quantities have their usual meaning. Further, prove that $\hat{n}.\bar{s}(\hat{n}') = \hat{n}'.\bar{s}(\hat{n})$ iff $\underset{\sim}{T}$ is symmetric. 8

b) The stress matrix at a point in a material is given by $[\tau_{ij}] = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$. Find the stress vector at P acting on the plane element parallel to the plane $x_1 + 2x_2 + 2x_3 = 0$. Also, find normal and shear stresses on the element. 8

PART – C

6. a) Derive the equation of continuity in the Eulerian form from its Lagrangian form. 4

b) The stress matrix in a continuum in equilibrium is given by

$$[\tau_{ij}] = \begin{bmatrix} x_1^2 & 2x_1x_2 & 0 \\ 2x_1x_2 & x_2^2 & 0 \\ 0 & 0 & x_1^2 + x_2^2 \end{bmatrix}.$$

Find the body force acting on the continuum. 5

c) Using the appropriate balance law, show that the stress tensor is symmetric. 7

7. a) Establish stress-strain relation for a linear isotropic elastic solid. 6

b) With usual notations, show that the change in volume of an elastic body in the absence of inertial effects is given by

$$\delta v = \frac{1-2\upsilon}{E}\left[\int_V f_i x_i dv + \int_S s_i x_i ds\right].$$ 5

c) Derive Navier's equation in its standard form. 5

8. a) Prove that every motion of an elastic fluid under conservative body force is circulation preserving. 5

b) Derive Navier-Stokes equation for a compressible fluid in its usual form. 5

c) The velocity field $\vec{v} = K(x_1^2 - x_2^2)\hat{e}_1 - 2K\, x_1 x_2\, \hat{e}_2$ (K = constant) satisfies the Navier-Stokes equation for an incompressible fluid in the absence of body force. Find the pressure distribution. 6

Second Semester M.Sc. Degree Examination, June/July 2014
(RNS) (2011-12 & Onwards)
MATHEMATICS
M-205 : Continuum Mechanics

Time : 3 Hours

Max. Marks : 80

Instructions : *Answer **any five** questions, choosing at least **one** from each Part. **All** questions carry **equal** marks.*

PART – A

1. a) Define the symbols δ_{ij} and ε_{ijk}. Show that $\varepsilon_{ijk}\varepsilon_{pqk} = \delta_{ip}\delta_{jq} - \delta_{iq}\delta_{jp}$. 4

b) If $\underset{\sim}{A}$ is an isotropic tensor of order 2, then prove that $\underset{\sim}{A} = \alpha \underset{\sim}{I}$ for some scalar α. 7

c) Show that a tensor $\underset{\sim}{A}$ is orthogonal iff $\underset{\sim}{A}\bar{a} . \underset{\sim}{A}\bar{b} = \bar{a} . \bar{b}$ for all vectors $\bar{a}$ and $\bar{b}$. Also, deduce that $|\underset{\sim}{A}\bar{a}| = |\bar{a}|$ if $\underset{\sim}{A}$ is orthogonal. 5

2. a) Define the dual vector of the tensor $\underset{\sim}{A}$. If $\bar{w}$ is the dual vector of a skew tensor $\underset{\sim}{A}$, then show that $|\bar{w}| = \frac{1}{\sqrt{2}}|\underset{\sim}{A}|$. 5

b) For the vector field $\bar{u} = x_1^2 x_2 \hat{e}_1 + x_2^2 x_3 \hat{e}_2 + x_3^2 x_1 \hat{e}_3$, verify the identity $\operatorname{div}(\nabla \bar{u}^T) = \nabla \operatorname{div} \bar{u}$. 6

c) State and prove divergence theorem for a tensor field $\underset{\sim}{A}$. 5

P.T.O.

PART – B

3. a) Write a short note on the following :

i) Continuum hypothesis

ii) Eulerian and Lagrangian descriptions of motion. 5

b) For the deformation defined by the equations

$x_1 = x_1^0 + x_2^0, x_2 = x_1^0 - 2x_2^0, x_3 = x_1^0 + x_2^0 - x_3^0,$

Find $\underset{\sim}{F}$, J and $\underset{\sim}{F}^{-1}$. Is the deformation isochoric ? 6

c) Define Green deformation tensor and Green strain tensor and find the expressions for the same in the material form. 5

4. a) Prove the following :

i) $\frac{D}{Dt}(\log j) = \text{div}\,\bar{v}$

ii) $\frac{D}{Dt}\int_v \phi dv = \int_v \frac{\partial \phi}{\partial t} dv + \int_s \phi \bar{v}.\hat{n} ds$

(6+5)

b) Show that a motion is circulation preserving iff $\frac{\partial \bar{w}}{\partial t} = \text{curl}\,(\bar{v} \times \bar{w})$, where $\bar{w} = \text{curl}\,\bar{v}$. 5

5. a) Establish Cauchy's law in its standard form. 5

b) The stress matrix at a point in a material is given by

$$[\tau_{ij}] = \begin{bmatrix} x_1x_3 & x_3^2 & 0 \\ x_3^2 & 0 & -x_2 \\ 0 & -x_2 & 0 \end{bmatrix}$$

Find the stress vector at the point (1, 0, –1) on the surface $x_1 = x_2^2 + x_3^2$. 6

c) Explain normal and shear stresses and obtain the relation between them. 5

PART – C

6. a) For every motion of an incompressible continuum, show that the following conditions are equivalent to one another

i) $\operatorname{div} \bar{v} = 0$

ii) $\frac{D\rho}{Dt} = 0$

iii) $\rho = \rho_0$

iv) $J = 1$ 6

b) For a certain flow of a continuum the velocity field is given by $v_i = x_i/(1+t)$. Show that the density at time t is $\rho = \rho_0/(1+t)^3$. Deduce that $\rho x_1 x_2 x_3 = \rho^0 x_1^0 x_2^0 x_3^0$. 5

c) With usual notations, derive Cauchy's equation of motion in the form $\operatorname{div} \underset{\sim}{T}^T + \rho\bar{b} = \rho\frac{D\bar{v}}{Dt}$. 5

7. a) For a linear isotropic elastic solid, show that 10

$\underset{\sim}{T} = \lambda(\operatorname{tr}\underset{\sim}{E})\underset{\sim}{I} + 2\mu\underset{\sim}{E}$

and $\underset{\sim}{E} = \frac{1}{2\mu}\left[\underset{\sim}{T} - \frac{\lambda}{3\lambda+2\mu}(\operatorname{tr}\underset{\sim}{T})\underset{\sim}{I}\right]$, where the quantities have their usual meaning. Further, show that $\operatorname{tr}\underset{\sim}{T} = (3\lambda+2\mu)\operatorname{tr}\underset{\sim}{E}$ and $\underset{\sim}{T}^{(d)} = 2\mu\underset{\sim}{E}^{(d)}$.

b) Derive Navier's equation of equilibrium in its standard form. 6

8. a) Derive the equation of motion for an elastic fluid. Integrate this equation considering the body force is conservative and the flow is of potential kind. 10

b) For a steady creeping flow of an incompressible viscous fluid under zero body force, show that $\nabla^2 p = 0$ and $\nabla^4 \psi = 0$, where ϕ is the pressure and ψ is a two-dimensional stream function. 6